

(i) Printed Pages : 3

Roll No.

(ii) Questions : 8

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Exam. Code :

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B.A./B.Sc. (General) 5th Semester
1128

MATHEMATICS
Paper-I : Analysis—I

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :- Attempt FIVE questions in all, selecting at least two questions from each Section. All questions carry equal marks.

SECTION—A

1. (a) Show that set of irrational numbers is uncountable.

(b) By considering the integral $\int_n^{n+1} \frac{1}{x} dx, n > 0$ prove that

$$\frac{1}{n+1} \leq \log\left(1 + \frac{1}{n}\right) \leq \frac{1}{n}.$$

2. (a) If f is integrable on $[a, b]$ and c is a real number then cf is

integrable on $[a, b]$. Moreover $\int_a^b cf dx = c \int_a^b f dx$.

(b) Proceeding from the definition, compute $\int_1^2 \frac{1}{x} dx$.

3. (a) State and prove fundamental theorem of Integral Calculus.

(b) Prove that $\int_{-1}^{\infty} \frac{x+1}{(x+2)^6} dx = \frac{1}{20}$.

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4. (a) Prove that $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$, $m > 0, n > 0$.

(b) Prove that $\int_0^{\infty} x^n e^{-ax^2} dx = \frac{1}{2a^{n+1}} \frac{\Gamma(n+1)}{2}$, where $n + 1 > 0$.

Hence evaluate $\int_{-\infty}^{\infty} e^{-a^2x^2} dx$.

SECTION—B

5. (a) Discuss the convergence of $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$.

(b) Show that improper integral $\int_0^1 \frac{dx}{\sqrt{x-x^2}}$ is convergent and its value is π .

6. (a) If $\phi(x)$ is bounded and monotonic in $[a, \infty)$ and tends to 0 as $x \rightarrow \infty$ and $\int_a^t f(x) dx$ is bounded for all $t \geq a$, then

$\int_a^{\infty} f(x) \phi(x) dx$ is convergent at ∞ .

(b) Use Abel's test to show that $\int_0^{\infty} e^{-ax} \frac{\sin x}{x} dx$, $a \geq 0$ is convergent.

7. (a) Show that $\int_0^{\frac{\pi}{2}} x^m \operatorname{cosec}^n x dx$ exist if and only if $n < m + 1$.

(b) Show that
$$\int_0^{\infty} \frac{ae^{-ce^{ax}}}{1-e^{-ax}} - \frac{be^{-ce^{bx}}}{1-e^{-bx}} dx = e^{-c} \log \frac{b}{a}$$

where $a, b, c > 0$

8. (a) Evaluate $\int_0^{\infty} \frac{e^{-ax} \sin bx}{x} dx$, where $a \geq 0$. Hence deduce that

$$\int_0^{\infty} \frac{\sin bx}{x} dx = \frac{\pi}{2}.$$

(b) Show that for $x^2 \leq 1$.

$$\int_0^{\infty} \log(1 - a^2 \cos^2 x) dx = \pi \log(1 + \sqrt{1 - a^2}) - \pi \log 2.$$

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